

Definition 1. Positive integer Exponent: $x^n = \overbrace{x \times x \cdots \times x}^{n\text{-times}}$, where $n \geq 0$ is called the exponent and x is called the base.

Example 1.

$$3^4 = 3 \times 3 \times 3 \times 3 = 81.$$

$5^0 = 1$, note that any number to power Zero is 1.

Definition 2. Negative integer Exponent: $x^{-n} = \frac{1}{x^n}$, where $n > 0$.

Example 2.

$$3^{-4} = \frac{1}{3^4} = \frac{1}{3 \times 3 \times 3 \times 3} = \frac{1}{81}.$$
$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}.$$

Definition 3. Product Rule: $x^m \times x^n = x^{m+n}$. "Note that this rule applies only for same base."

Example 3.

$$3^4 \times 3 = 3^{4+1} = 243.$$
$$5^3 \times 5^{-4} = 5^{3+(-4)} = 5^{3-4} = 5^{-1} = \frac{1}{5}.$$

Definition 4. Quotient Rule: $\frac{x^m}{x^n} = x^{m-n}$. "Again note here the same base."

Example 4.

$$\frac{3^4}{3} = 3^{4-1} = 27.$$
$$\frac{5^3}{5^{-4}} = 5^{3-(-4)} = 5^{3+4} = 78125.$$

Definition 5. Power Rule: $(x^m)^n = x^{m \times n}$

Example 5.

$$(3^4)^2 = 3^{4 \times 2} = 3^8 = 6561.$$
$$(5^3)^{-2} = 5^{3 \times (-2)} = 5^{-6} = \frac{1}{5^6} = \frac{1}{15625}.$$

Definition 6. Power-of-a-Product Rule: $(x \times y)^m = x^m \times y^m$.

Example 6.

$$(3 \times 5)^2 = 3^2 \times 5^2 = 9 \times 25 = 225.$$
$$\left(\frac{5^3}{5}\right)^2 = (5^{3-1})^2 = 5^{2 \times 2} = 5^4 = 625,$$

or

$$\left(\frac{5^3}{5}\right)^2 = \frac{5^{3 \times 2}}{5^{1 \times 2}} = \frac{5^6}{5^2} = 5^{6-2} = 625,$$

Example 7. Simplify:

$$1. \frac{x^2 y^{-2}}{4x^{-1} y^2} = \frac{x^{2-(-1)} y^{(-2)-2}}{4} = \frac{x^{2+1} y^{-4}}{4} = \frac{x^3}{4y^4}.$$

$$2. \frac{27a^{-3} b^5}{9a^{-4} b^7} = \frac{3a^{(-3)-(-4)} b^{(5-7)}}{1} = 3a^{(-3+4)} b^{-2} = \frac{3a}{b^2}.$$

Definition 7. Scientific Notation: *Scientific notation of a number has the form*

$$c \times 10^n,$$

where c is a real number in decimal notation with $1 \leq c < 10$ and n is an integer.

I. How to Convert a Decimal Number to Scientific Notation?

1. Count the number, n , of places the decimal point in the given number must be moved to obtain a number c with $1 \leq c < 10$.
2. If the decimal point is moved n places to the left, then the scientific notation is $c \times 10^n$. If the decimal point is moved n places to the right, then the scientific notation is $c \times 10^{-n}$.
3. If the decimal point does need to be moved, then the scientific notation is $c \times 10^0$.

Example 8. Write each decimal number in scientific notation:

$$1. 421,000 = 421000.0 = 4 \overset{5\text{-places}}{\overbrace{21000}^{\leftarrow}} = 4.21 \times 10^5.$$

$$2. 3.621 = 3.621 \times 10^0.$$

$$3. 0.000561 = 0 \overset{4\text{-places}}{\overbrace{.000561}^{\rightarrow}} = 5.61 \times 10^{-4}.$$